Algebra 1

Semester 1
Common Core State Standards Initiative

COURSE DESCRIPTION

Algebra 1 Semester 2

Will meet graduation requirements for Algebra 1 Semester 2

Subject Area: Mathematics

Course Number: 1200310

Course Title: Algebra 1 Semester 1

Credit: 0.5
Algebra 1

Introduction

American Worldwide Academy’s math course, AWA Algebra 1, focuses on the fundamental skills that are necessary for understanding the basics of algebra. This Study guide addresses essential standards of mathematics, such as number sense, algebra, and graph analysis. AWA Algebra 1 is full of practical, useful information geared to helping students recover credit for algebra while mastering the basics. This Study guide will be helpful to any student who has previously had difficulties with understanding algebraic concepts and skills.
Course Objectives

After successful completion of this course, students will know and be able to do the following:

Algebra Standards and Concepts

Section 1: Real and Complex Number Systems -
Students expand and deepen their understanding of real and complex numbers by comparing expressions and performing arithmetic computations, especially those involving square roots and exponents. They use the properties of real numbers to simplify algebraic expressions and equations, and they convert between different measurement units using dimensional analysis.

- Know equivalent forms of real numbers (including integer exponents and radicals, percents, scientific notation, absolute value, rational numbers, irrational numbers).
- Compare real number expressions.
- Simplify real number expressions using the laws of exponents.
- Perform operations on real numbers (including integer exponents, radicals, percents, scientific notation, absolute value, rational numbers, and irrational numbers) using multi-step and real-world problems.
- Use dimensional (unit) analysis to perform conversions between units of measure, including rates.
- Identify the real and imaginary parts of complex numbers and perform basic operations.
- Represent complex numbers geometrically.
- Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Section 2: Relations and Functions –
Students draw and interpret graphs of relations. They understand the notation and concept of a function, find domains and ranges, and link equations to functions.

- Create a graph to represent a real-world situation.
- Interpret a graph representing a real-world situation.
- Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions.
- Determine the domain and range of a relation.
- Graph absolute value equations and inequalities in two variables.
- Identify and graph common functions (including but not limited to linear, rational, quadratic, cubic, radical, absolute value).
- Perform operations (addition, subtraction, division and multiplication) of functions algebraically, numerically, and graphically.
- Determine the composition of functions.
- Recognize, interpret, and graph functions defined piece-wise, with and without technology.
- Describe and graph transformations of functions.
- Solve problems involving functions and their inverses.
- Solve problems using direct, inverse, and joint variations.
- Solve real-world problems involving relations and functions.
Getting Started

You will learn much from this course that will help you in your future studies and career. In addition to reviewing and completing the study guide and textbook, your Final Examination will be evidence that you have mastered the standards for algebra. You will know the concepts and be able to do the skills that will earn you one full credit for Algebra 1.

If you are ready to begin, turn to the next page in this Study guide: the Progress Chart and Self-Test Schedule, which will serve as a guide to help you move through the course. Let’s get started on earning that algebra credit—good luck!
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Algebra 1

Content Review for Section 1: Real Number System

Before you begin learning, relearning, or reviewing algebra, you need to feel comfortable with some pre-algebra terms and operations. The first items you should become familiar with are the different categories or types of numbers and the common math symbols.

Categories of Numbers

Number Sets

In doing algebra, you work with several categories of numbers.

- **Natural or counting numbers.** The numbers 1, 2, 3, 4, ... are called natural or counting numbers.
- **Whole numbers.** The numbers 0, 1, 2, 3, ... are called whole numbers.
- **Integers.** The numbers −2, −1, 0, 1, 2, ... are called integers.
- **Negative integers.** The numbers −3, −2, −1 are called negative integers.
- **Positive integers.** The natural numbers are sometimes called the positive integers.
- **Rational numbers.** Fractions, such as $\frac{3}{2}$ or $\frac{7}{8}$, are called rational numbers. Since a number such as 5 may be written as $\frac{5}{1}$, all integers are rational numbers. All rational numbers can be written as fractions $\frac{a}{b}$, with $a$ being an integer and $b$ being a natural number. Terminating and repeating decimals are also rational numbers, because they can be written as fractions in this form.
- **Irrational numbers.** Another type of number is an irrational number. Irrational numbers cannot be written as fractions $\frac{a}{b}$, with $a$ being an integer and $b$ being a natural number. $\sqrt{3}$ and $\pi$ are examples of irrational numbers. An irrational number, when exactly expressed as a decimal, neither terminates nor has a repeating decimal pattern.
- **Even numbers.** Even numbers are integers divisible by 2: ... −6, −4, −2, 0, 2, 4, 6, ...
- **Prime numbers.** A prime number is a natural number that has exactly two different factors, or that can be perfectly divided by only itself and 1. For example, 19 is a prime number because it can be perfectly divided by only 19 and 1, but 21 is not a prime number because 21 can be perfectly divided by other numbers (3 and 7). The only even prime number is 2; thereafter, any even number may be divided perfectly by 2. Zero and 1 are not prime numbers or composite numbers. The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.
- **Odd numbers.** Odd numbers are integers not divisible by 2: ... −5, −3, −1, 1, 3, 5, ...
- **Composite numbers.** A composite number is a natural number divisible by more than just 1 and itself: ...4, 6, 8, 9,...
• **Squares.** Squares are the result when numbers are multiplied by themselves, that is, raised to the second power. $2 \cdot 2 = 4; \ 3 \cdot 3 = 9$. The first six squares of natural numbers are 1, 4, 9, 16, 25, 36.

• **Cubes.** Cubes are the result when numbers are multiplied by themselves and then again by the original number, that is, raised to the third power. $2 \cdot 2 \cdot 2 = 8; \ 3 \cdot 3 \cdot 3 = 27$. The first six cubes of natural numbers are 1, 8, 27, 64, 125, 216.

**Common Math Symbols**

The following math symbols appear throughout algebra. Be sure to know what each symbol represents.

Symbol references:

- $\neq$ is not equal to
- $>$ is greater than
- $<$ is less than
- $\geq$ is greater than or equal to (also written $\geq$)
- $\leq$ is less than or equal to (also written $\leq$)
- $\approx$ is approximately equal to (also $\approx$)
- $\neq$ is not greater than
- $<$ is not less than
- $\neq$ is not greater than or equal to
- $\leq$ is not less than or equal to
- $\approx$ is approximately equal to

**Properties of Operations**

A **number property** states a relationship between numbers and expressions.

The **commutative properties of addition and multiplication** state that the order in which numbers are added (+) or multiplied (*) does not matter.

$$a + b = b + a \quad \text{and} \quad a \times b = b \times a$$

The **associative properties of addition and multiplication** state that the way in which numbers are grouped when more than two numbers are added or multiplied does not matter.

$$(a + b) + c = a = (b + c) \quad \text{and} \quad (a \times b) \times c = a \times (b \times c)$$

The **distributive property** relates multiplication to addition or subtraction. This property states that everything inside parentheses is multiplied by what is outside the parentheses.

$$a(b + c) = a \times b + a \times c \quad \text{and} \quad a(b - c) = a \times b - a \times c$$
Laws of Exponents

Exponents are also called **Powers** or **Indices**

The exponent of a number says **how many times** to use the number in a **multiplication**.

In this example: \( 8^2 = 8 \times 8 = 64 \)

- In words: \( 8^2 \) could be called "8 to the second power", "8 to the power 2" or simply "8 squared"

So an Exponent just saves you writing out lots of multiplies!

**Example:** \( a^7 = a \times a \times a \times a \times a \times a \times a = aaaaaa \)

Notice how I just wrote the letters together to mean multiply? We will do that a lot here.

**The Key to the Laws**

Writing all the letters down is the key to understanding the Laws

**Example:** \( x^2 x^3 = (xx)(xxx) = xxxxx = x^5 \)

Which shows that \( x^2 x^3 = x^5 \), but more on that later!

So, when in doubt, just remember to write down all the letters (as many as the exponent tells you to) and see if you can make sense of it.

All you need to know ...

The "Laws of Exponents" (also called "Rules of Exponents") come from three ideas:

- The exponent says **how many times** to use the number in a multiplication.
- A **negative exponent** means **divide**, because the opposite of multiplying is dividing
- A **fractional exponent** like \( 1/n \) means to **take the nth root**: \( x^{1/n} = \sqrt[n]{x} \)

If you understand those, then you understand exponents! And all the laws below are based on those ideas.
Here are the Laws (explanations follow):

<table>
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<td>( x^1 = x )</td>
<td>( 6^1 = 6 )</td>
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<td>( x^0 = 1 )</td>
<td>( 7^0 = 1 )</td>
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<tr>
<td>( x^{-1} = 1/x )</td>
<td>( 4^{-1} = 1/4 )</td>
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<tr>
<td>( x^m x^n = x^{m+n} )</td>
<td>( x^2 x^3 = x^{2+3} = x^5 )</td>
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<td>( x^m/x^n = x^{m-n} )</td>
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<tr>
<td>((x^m)^n = x^{mn})</td>
<td>((x^2)^3 = x^{2\times3} = x^6)</td>
</tr>
<tr>
<td>((xy)^n = x^n y^n)</td>
<td>((xy)^3 = x^3 y^3)</td>
</tr>
<tr>
<td>((x/y)^n = x^n / y^n)</td>
<td>((x/y)^2 = x^2 / y^2)</td>
</tr>
<tr>
<td>(x^n = 1/x^n)</td>
<td>(x^3 = 1/x^3)</td>
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And the law about Fractional Exponents:

\[
x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m
\]

\[
x^{\frac{2}{3}} = \sqrt[3]{x^2}
\]

**Laws Explained**

The first three laws above (\( x^1 = x \), \( x^0 = 1 \) and \( x^{-1} = 1/x \)) are just part of the natural sequence of exponents. Have a look at this:

**Example: Powers of 5**

.. etc..

| \( 5^2 \) | \( 1 \times 5 \times 5 \) | 25 |
| \( 5^1 \) | \( 1 \times 5 \) | 5 |
| \( 5^0 \) | 1 | 1 |
| \( 5^{-1} \) | \( 1 / 5 \) | 0.2 |
| \( 5^{-2} \) | \( 1 / 5 / 5 \) | 0.04 |

.. etc..

Look at that table for a while ... notice that positive, zero or negative exponents are really part of the same pattern, i.e. 5 times larger (or 5 times smaller) depending on whether the exponent gets larger (or smaller).
Algebra 1

The law that $x^m \times x^n = x^{m+n}$

With $x^m \times x^n$, how many times will you end up multiplying "x"? Answer: first "m" times, then **by another** "n" times, for a total of "m+n" times.

Example: $x^2 \times x^3 = (xx)(xxx) = xxxxx = x^5$

So, $x^2 \times x^3 = x^{(2+3)} = x^5$

**The law that $x^m / x^n = x^{m-n}$**

Like the previous example, how many times will you end up multiplying "x"? Answer: "m" times, then **reduce that** by "n" times (because you are dividing), for a total of "m-n" times.

Example: $x^4 / x^2 = (xxxx) / (xx) = xx = x^2$

So, $x^4 / x^2 = x^{(4-2)} = x^2$

(Remember that $x/x = 1$, so every time you see an x "above the line" and one "below the line" you can cancel them out.)

This law can also show you why $x^0 = 1$:

Example: $x^2 / x^2 = x^{2-2} = x^0 = 1$

**The law that $(x^m)^n = x^{mn}$**

First you multiply "m" times. Then you have **to do that** "n" times, for a total of m×n times.

Example: $(x^3)^4 = (xxx)(xxx)(xxx)(xxx)(xxx) = xxxxxxxxxx = x^{12}$

So $(x^3)^4 = x^{3\times 4} = x^{12}$

**The law that $(xy)^n = x^n y^n$**

To show how this one works, just think of re-arranging all the "x"s and "y" as in this example:

Example: $(xy)^3 = (xy)(xy)(xy) = xyxyxy = xxxyyy = (xxx)(yyy) = x^3 y^3$

**The law that $(x/y)^n = x^n / y^n$**

Similar to the previous example, just re-arrange the "x"s and "y"s

Example: $(x/y)^3 = (x/y)(x/y)(x/y) = (xxx)/(yyy) = x^3 / y^3$
The law that $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

OK, this one is a little more complicated!

I suggest you read Fractional Exponents first, or this may not make sense.

Anyway, the important idea is that:

$$x^{\frac{1}{n}} = \text{The n-th Root of } x$$

And so a fractional exponent like $4^{3/2}$ is really saying to do a cube (3) and a square root (1/2), in any order.

Just remember from fractions that $\frac{m}{n} = m \times \frac{1}{n}$:

Example:

$$x^{\frac{m}{n}} = x^{(m \times \frac{1}{n})} = \left(x^m\right)^{\frac{1}{n}} = \frac{n}{\sqrt[n]{x^m}}$$

Numbers and Number Operations

A. Definitions

1. **Whole Numbers** - are positive numbers beginning with zero such as 0, 1, 2, 3, 4, . . .

2. **Fraction** - is a part of a whole number written with a fraction bar where the top number is the numerator and the bottom number is the denominator.
   - Given the fraction $\frac{3}{5}$, 3 is the numerator and 5 is the denominator.
   - To change a fraction to a decimal, divide the denominator into the numerator.
     
     for example: $\frac{1}{2} = 1 \div 2 = 0.5$

3. **Decimal** - is a part of a whole number written with a decimal point where the place values to the right of the decimal are: . (tenths) (hundredths) (thousandths) etc.
   - To round a number to a particular place value, look at the number to the immediate right of the place value and if it is 5 or more round up one, but if it is 4 or less leave at the same value.

Examples:

- 2.4372 rounded to the nearest 100th – **2.44**
- 0.5612 rounded to the nearest 1000th – **0.561**
B. To change a decimal to a percent, move the decimal two places to the right.

Examples:
- 0.5 – 50%
- 0.467 – 46.7%

4. Percent - some part out of 100.

- To change a percent to a fraction, put the number over 100 and then reduce.

Example:
- 50% = \frac{50}{100} = .5

- To change a percent to a decimal, move the decimal two places left.

Examples:
- 65% = .65
- 41.8% = .418

❖ Order of Operations

The order of operations is used to simply numerical and algebraic expressions. It sets out the steps to follow in order:

P.E.M.D.A.S.

1. Simplify all expressions in parentheses, including brackets and absolute value signs.
2. Simplify all expressions that have a square root or an exponent.
3. Perform multiplication and division in order, from left to right.
4. Perform addition and subtraction in order, from left to right.

Problem-Solving Strategies for Word Problems

The following steps are helpful in solving applications, also called word problems:

1. Take time to study the problem.
2. Evaluate the information given.
3. Select a strategy for solving the problem.
4. Set up the problem and estimate a reasonable answer.
5. Find the answer and check it.
Algebra 1

Before we begin simplifying problems using the Order of Operations, let's examine how failure to use the Order of Operations can result in a wrong answer to a problem.

\[ 2 + 5 \cdot x \]

Without the Order of Operations one might decide to simplify the problem working left to right. He or she would add two and five to get seven, then multiply seven by \( x \) to get a final answer of \( 7x \). Another person might decide to make the problem a little easier by multiplying first. He or she would have first multiplied 5 by \( x \) to get \( 5x \) and then found that you can't add 2 and \( 5x \) so his or her final answer would be \( 2 + 5x \). Without a standard like the Order of Operations, a problem can be interpreted many different ways.

**Example**

\[ 3 \times (5 + 8) - 22 / 4 + 3 \]

Parenthesis first: \( 5 + 8 = 13 \)

\[ 3 \times 13 - 22 / 4 + 3 \]

Exponent next: square the 2 or \( 22 = 4 \)

\[ 3 \times 13 - 4 / 4 + 3 \]

Multiplication and Division next (\( 3 \times 13 \) \( 4 / 4 \))

*left to right:*

\[ 39 - 1 + 3 \]

Addition and Subtraction next

*left to right:*

\[ 39 - 1 + 3 = 41 \]

Note that we first subtracted 1 from 39 (left to right!), then added the 3 for the correct answer, 41.
Algebra 1

Unit Conversion

"Well let's see, there's 3 feet in one yard so there must be 6 feet in 2 yards." That works just fine.

But what if you wanted to find out how many inches there are in 2,500 miles. Or find out how many feet per second there are in 55 miles per hour. Then we need a better plan.

Good news! We have one. Here it is:

Take another look at that "How many feet are in 2 yards?" problem.

We have 2 yards, and we want to have our answer in feet. To get there, we need to do something very, very tricky. We need to multiply … by 1.

OK, OK, there really IS a trick. The trick is the way we write the number 1 that we use to multiply. Any time we have a fraction with the same amount on the top and the bottom we have a fraction equal to 1 (except when that amount is zero).

Anyway, if we have a fraction with 1 yard on the top and 1 yard on the bottom, we have a fraction equal to 1:

\[
\frac{1 \text{ Yard}}{1 \text{ Yard}} = 1
\]

We also know that 1 yard is equal to 3 feet. So instead of 1 yard, we can write 3 feet for either the top part or the bottom part of our fraction and we haven't changed anything.

Like this:

\[
\frac{3 \text{ Feet}}{1 \text{ Yard}} = 1
\]

The trick is the way we write the fraction that is equal to 1. We write it with things that LOOK different on the top and the bottom, but are actually worth the same amount. Now we have our new special name for 1. We are ready to multiply the 2 yards:

\[
2 \text{ Yards} \times \frac{3 \text{ Feet}}{1 \text{ Yard}} =
\]

To make things even out, we can put a denominator of 1 under the 2 Yards.

\[
\frac{2 \text{ Yards}}{1} \times \frac{3 \text{ Feet}}{1 \text{ Yard}} = \frac{6 \text{ Yards} \times \text{ Feet}}{1 \text{ Yard}}
\]
Algebra 1

OK, We did it, but look! What kind of a thing is "6 Yards x Feet" ??? Who Knows! And if we had to leave it looking like that we wouldn't have anything too great. But look! We have the word yards in the top and the word yard (close enough) in the bottom part.

We can cancel them out. Works every time.

The only thing is that everything on top and everything on the bottom must be multiplied. No addition or subtraction. So...

\[
\frac{6 \text{ Yard}}{1 \text{ Yard}} \times \frac{\text{ 6 Feet}}{1} = 6 \text{ Feet}
\]

That means: 2 Yards = 6 Feet

Hey, we KNEW that 6 Feet was the answer all along. So this is no shock. But let's go over exactly how we come up with the funny name for 1. The fraction gets units that are what we have to begin with and what we want to end up with. We had yards and wanted to get to feet. We know we want feet for 1 number and yards for the other. Which gets which? Do we want:

Here's the deal. Put the units you want on top and the units you have on the bottom. The units you have go on the bottom so you can cancel them! We had yards to begin with, so the one we want is:

\[
\frac{3 \text{ Feet}}{1 \text{ Yard}}
\]

Example:

Suppose we're really bored and want to find out how many feet there are in 2500 miles. You look up (or maybe you know) that there are 5,280 feet in a mile. That is: 5,280 Feet = 1 Mile

Since these 2 things are worth the same amount, we can use them in a fraction as our names for 1. We have miles and want feet, so miles goes on the bottom and feet goes on the top:

\[
\frac{5,280 \text{ Feet}}{1 \text{ Mile}} = 1
\]

Now we multiply 2500 miles by this fraction that's equal to 1. So everybody has a denominator, we put a 1 under the 2500 miles

\[
\frac{2,500 \text{ Miles}}{1} \times \frac{5,280 \text{ Feet}}{1 \text{ Mile}} = \frac{13,200,000 \text{ Miles} \times \text{ Feet}}{1 \text{ Mile}}
\]

Since we have miles on the top and mile on the bottom, and everything is multiplied, we can get rid of them.

\[
\frac{13,200,000 \text{ Miles} \times \text{ Feet}}{1 \text{ Mile}} = \frac{13,200,000 \text{ Feet}}{1} = 13,200,000 \text{ Feet}
\]
Example:

Tennessee Ernie Ford asks you to change 16 tons to ounces. If you look in a reference book, you will find that: 1 ton = 2000 pounds. You will also find that 1 pound = 16 ounces. But you probably won't find how many ounces are in a ton. So here's the plan. First we change 16 tons to some number of pounds. Then we change that number of pounds to ounces.

OK, we have 16 tons and want pounds. That means pounds goes on the top and ounces goes on the bottom. 2,000 Pounds = 1 Ton.

\[
\frac{16 \text{ Tons}}{1} \times \frac{2,000 \text{ Pounds}}{1 \text{ Ton}} = \frac{32,000 \text{ Tons} \times \text{ Pounds}}{1 \text{ Ton}}
\]

We have Tons on the top and Ton on the bottom. Everything is multiplied. So we can cancel tons.

\[
\frac{32,000 \text{ Tons} \times \text{ Pounds}}{1 \text{ Ton}} = \frac{32,000 \text{ Pounds}}{1} = 32,000 \text{ Pounds}
\]

One part down and one to go. Now we have pounds and want ounces. So 16 Ounces goes on top and 1 Pound goes on the bottom. 1 Pound = 16 Ounces.

\[
\frac{32,000 \text{ Pounds}}{1} \times \frac{16 \text{ Ounces}}{1 \text{ Pound}} = \frac{512,000 \text{ Pounds} \times \text{ Ounces}}{1 \text{ Pound}}
\]

We have Pounds on the top and Pounds on the bottom. Everything is multiplied. We can cancel the Pounds.

\[
\frac{512,000 \text{ Pounds} \times \text{ Ounces}}{1 \text{ Pound}} = \frac{512,000 \text{ Ounces}}{1} = 512,000 \text{ Ounces}
\]

\[\text{Zero Product Property}\]

The "Zero Product Property" says that: if \(a \times b = 0\) then \(a = 0\) or \(b = 0\) (or both \(a=0\) and \(b=0\)).

It can help you solve equations:

Example: Solve \((x-5)(x-3) = 0\)

The "Zero Product Property" says:

If \((x-5)(x-3) = 0\) then \((x-5) = 0\) \text{ or } (x-3) = 0

Now we just solve each of those:

For \((x-5) = 0\) we get \(x = 5\); for \((x-3) = 0\) we get \(x = 3\)

And the solutions are: \(x = 5\), or \(x = 3\).
Algebra 1

Standard Form of an Equation

Sometimes you can solve an equation by putting it into Standard Form and then using the Zero Product Property:

The "Standard Form" of an equation is: \((\text{some expression}) = 0\)

In other words, "\(= 0\)" is on the right, and everything else is on the left.

Example: Put \(x^2 = 7\) into Standard Form

Answer: \(x^2 - 7 = 0\)

Standard Form and the Zero Product Property

Example: Solve \(5(x+3) = 5x(x+3)\)

It is tempting to divide by \((x+3)\), but that would be dividing by zero when \(x = -3\)

So instead we can use "Standard Form":

\[ 5(x+3) - 5x(x+3) = 0 \]

Which can be simplified to:

\[ (5-5x)(x+3) = 0 \]

\[ 5(1-x)(x+3) = 0 \]

Then the "Zero Product Property" says:

\[ (1-x) = 0, \text{ or } (x+3) = 0 \]

And the solutions are:

\[ x = 1, \text{ or } x = -3 \]

Example: Solve \(x^3 = 25x\)

It is tempting to divide by \(x\), but that would be dividing by zero when \(x = 0\)

So let's use Standard Form and the Zero Product Property.
Algebra 1

Bring all to the left hand side:

\[ x^3 - 25x = 0 \]

Factor out \( x \):

\[ x(x^2 - 25) = 0 \]

\( x^2 - 25 \) is a difference of squares, and can be factored into \((x - 5)(x + 5)\):

\[ x(x - 5)(x + 5) = 0 \]

Now we can see three possible ways it could end up as zero:

\[ x = 0, \text{ or } x = 5, \text{ or } x = -5 \]
**Content Review for Section 2: Relations and Functions**

**Ordered Pairs**

An ordered pair is a pair of numbers in a specific order. For example, (1, 2) and (-4, 12) are ordered pairs. The order of the two numbers is important: (1, 2) is not equivalent to (2, 1) -- (1, 2) ≠ (2, 1).

**Using Ordered Pairs to Represent Variables**

Ordered pairs are often used to represent two variables. When we write \((x, y) = (7, -2)\), we mean \(x = 7\) and \(y = -2\). The number which corresponds to the value of \(x\) is called the \(x\)-coordinate and the number which corresponds to the value of \(y\) is called the \(y\)-coordinate.

**Example 1.** If \((x, y) = (-1, 4)\), what is the value of \(3x + 2y - 4\) ?

\[
3x + 2y - 4 = 3(-1) + 2(4) - 4 = -3 + 8 - 4 = 1
\]

**Example 2.** Which of the following ordered pairs \((x, y)\) are solutions to the equation \(\frac{2x-1}{y} - 6 = 1\) ? \{(4, 1),(-3, 1),(-3, 1),(-1, 1),(-3, -1),(-1, 4)\}

\((x, y) = (4, 1)\) : \(\frac{2x-1}{y} - 6 = \frac{2(4)-1}{1} - 6 = 7 - 6 = 1\). Solution.

\((x, y) = (5, 2)\) : \(\frac{2x-1}{y} - 6 = \frac{2(5)-1}{2} - 6 = \frac{9}{2} - 6 = \frac{3}{2} 1\). Not a solution.

\((x, y) = (-3, -1)\) : \(\frac{2x-1}{y} - 6 = \frac{2(-3)-1}{-1} - 6 = -7 - 6 = -13 1\). Not a solution.

\((x, y) = (-3, -1)\) : \(\frac{2x-1}{y} - 6 = \frac{2(1)-1}{4} - 6 = \frac{1}{4} - 6 = \frac{23}{4} 1\). Not a solution.

Thus, \{(4, 1),(-3, -1)\} are solutions to \(\frac{2x-1}{y} - 6 = 1\).

**Graphing Ordered Pairs**

We have graphed values on the number line in pre-algebra and in earlier chapters of algebra. However, we can only graph points of one variable on the number line; thus, we need a 2-dimensional (2 variable) way of representing points -- the \(xy\)-graph:

![xy-graph](image)
The horizontal axis, called the x-axis, represents values of x, and the vertical axis, called the y-axis, represents values of y. From now on, the word "graph" will refer to the xy-graph, unless specified otherwise.

To graph a point on the xy-graph, first find the x-coordinate on the x-axis. Then move up on the graph the number of spaces which is equal to the y-coordinate (or move down if the y-coordinate is negative). For example, to graph (2, 3), find 2 on the x-axis. Then move up 3 spaces. To graph (-2, 1), find -2 on the x-axis, then move up 1 space. To graph (1.5, -1), find 1.5 on the x-axis, then move down 1 space.

The point (0, 0) -- at the center of the graph -- is called the origin.

**Graphing Equations Using a Data Table**

One of the main uses of an xy-graph is to graph equations. If an equation has both an x and y variable, then it often has multiple solutions of the form (x, y). In fact, there are generally infinitely many solutions to an equation with two variables.

The solutions to an equation in two variables can be represented by a curve on an xy-graph; every point on the curve has coordinates which satisfy the equation. In fact, for linear equations (our only concern in this chapter), the curve representing the solutions to the equation will actually be a straight line.

*Example.* Here is the graph of 2y - x = 4:

If we pick any point on the line -- (2, 1), (-4, 0), or (1.5, 2 1/4), for example -- it will satisfy the equation 2y - x = 4. Try a few points; they need not have integer values.
Algebra 1

Making Data Tables

One way to graph an equation is by use of a data table. A data table is a list of \(x\)-values and their corresponding \(y\)-values. To make a data table, draw two columns. Label one column \(x\) and the other column \(y\). Then list the \(x\)-values -2, -1, 0, 1, 2 in the \(x\) column:

\[
\begin{array}{c|c|c|c|c}
\hline
x & -2 & -1 & 0 & 1 & 2 \\
\hline
y & & & & & \\
\end{array}
\]

Next, plug each value of \(x\) into the equation and solve for \(y\). Insert these values of \(y\) into the table, under their corresponding \(x\) values. For this example, we will use the equation \(2x - 4 = 3y\):

\[
\begin{align*}
x = -2 & : 2(-2) - 4 = 3y \Rightarrow 3y = -8 \Rightarrow y = -\frac{8}{3} \\
x = -1 & : 2(-1) - 4 = 3y \Rightarrow 3y = -6 \Rightarrow y = -2 \\
x = 0 & : 2(0) - 4 = 3y \Rightarrow 3y = -4 \Rightarrow y = -\frac{4}{3} \\
x = 1 & : 2(1) - 4 = 3y \Rightarrow 3y = -2 \Rightarrow y = -\frac{2}{3} \\
x = 2 & : 2(2) - 4 = 3y \Rightarrow 3y = 0 \Rightarrow y = 0 \\
\end{align*}
\]

Thus, the data table looks like:

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & -2 & -1 & 0 & 1 & 2 \\
\hline
y & -\frac{8}{3} & -2 & -\frac{4}{3} & -\frac{2}{3} & 0 \\
\end{array}
\]

Making Graphs Using Data Tables

To make a graph using the data table, simply plot all the points and connect them with a straight line. Extend the line on both sides and add arrows. This is to show that the line continues infinitely, even after it can be seen on the graph. Here is our sample data table as a graph:

Graph of \(2x - 4 = 3y\)

Note that the large dots on the line are unnecessary -- they are merely there to show the data points we plotted.
To check, pick a data point that is on the line but not in the chart -- it should satisfy the equation.

Notice also that it is not necessary to make a huge data table to graph a linear equation effectively. There is only one line through any two points, so already if one plots three points from a data table the redundancy of the third point acts as a check of the calculations. Of course, for more general equations whose graph does not consist of a straight line, more points are necessary to get an idea of the appearance of the graph.

Functions

Defining a function

The relation in Example has pairs of coordinates with unique first terms. When the x value of each pair of coordinates is different, the relation is called a function. A function is a relation in which each member of the domain is paired with exactly one element of the range. All functions are relations, but not all relations are functions. A good example of a functional relation can be seen in the linear equation \( y = x + 1 \), graphed in Figure 3. The domain and range of this function are both the set of real numbers, and the relation is a function because for any value of \( x \) there is a unique value of \( y \).

Graphs of functions

In each case in Figure 4 (a), (b), and (c), for any value of \( x \), there is only one value for \( y \). Contrast this with the graphs in Figure 5.
Graphs of relationships that are not functions

In each of the relations in Figure 5 (a), (b), and (c), a single value of \( x \) is associated with two or more values of \( y \). These relations are not functions.

![Figure 5. Graphs of relations that are not functions.](image)

The domain is the set of all first elements of ordered pairs (\( x \)-coordinates). The range is the set of all second elements of ordered pairs (\( y \)-coordinates).

Domain and range can be seen clearly from a graph.

<table>
<thead>
<tr>
<th>Example 1:</th>
<th>Example 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>{(3, 5), (4, 2), (6, -2), (1, 5)}</td>
<td>( y = x^2 + 2 )</td>
</tr>
<tr>
<td>Domain: {1, 3, 4, 6}</td>
<td>Domain: ( \mathbb{R} ) (all real numbers)</td>
</tr>
<tr>
<td>Range: {-2, 2, 5}</td>
<td>Range: ( y \geq 2 )</td>
</tr>
</tbody>
</table>

![Example 1](image)

![Example 2](image)

The two examples shown above are functions. But, as we know, not all graphs are functions.
The graph at the left is: \( f(x) = \pm \sqrt{x} \)

Since the graph **FAILS** the Vertical Line Test, this relation is not a function.

If we **restrict the graph** to only the "positive" (or we could have chosen negative) \( y \)-values, the graph will be a function:

\[ f(x) = +\sqrt{x} \]

In a similar fashion, we can also **restrict domains** to ensure that graphs are functions.

The graph at the left is:

If the domain for this graph is listed as "all Real numbers", this relation is **NOT** a function. At first glance this graph appears to pass the Vertical Line Test, but it is actually undefined at \( x = -1 \).

If we **restrict the domain** to be "all Real numbers excluding -1", our relation will be a function.

**Domain:** \( \mathbb{R} \setminus \{-1\} \)
Operations with Functions

You can add, subtract, multiply and divide functions!

The result will be a new function.

Let us try doing those operations on \( f(x) \) and \( g(x) \):

**Addition**

You can add two functions:

\[
(f+g)(x) = f(x) + g(x)
\]

*Note: I put the \( f+g \) inside () so you know they both work on \( x \).*

**Example:** \( f(x) = 2x+3 \) and \( g(x) = x^2 \)

\[
(f+g)(x) = (2x+3) + (x^2) = x^2+2x+3
\]

Sometimes you may need to combine like terms:

**Example:** \( v(x) = 5x+1, \ w(x) = 3x-2 \)

\[
(v+w)(x) = (5x+1) + (3x-2) = 8x-1
\]

The only other thing to worry about is the Domain (the set of numbers that go into the function), but I will talk about that later!

**Subtraction**

You can also subtract two functions:

\[
(f-g)(x) = f(x) - g(x)
\]

**Example:** \( f(x) = 2x+3 \) and \( g(x) = x^2 \)

\[
(f-g)(x) = (2x+3) - (x^2)
\]
You can multiply two functions:

\[(f \cdot g)(x) = f(x) \cdot g(x)\]

**Example:** \(f(x) = 2x + 3\) and \(g(x) = x^2\)

\[(f \cdot g)(x) = (2x + 3)(x^2) = 2x^3 + 3x^2\]

**Division**

And you can divide two functions:

\[(f/g)(x) = f(x) / g(x)\]

**Example:** \(f(x) = 2x + 3\) and \(g(x) = x^2\)

\[(f/g)(x) = (2x + 3)/x^2\]

**Function Composition**

**Example:** \(f(x) = x + 1/x\) and \(g(x) = x^2\)

\[(g \circ f)(x) = g(f(x)) = g(x + 1/x) = (x + 1/x)^2\]

**Domain**

It has been very easy so far, but now you must consider the **Domains** of the functions.

The domain is the set of all the values that go into a function.

The function must work for all values you give it, so it is up to you to make sure you get the domain correct!
Example: the domain for \( \sqrt{x} \) (the square root of \( x \))

You cannot have the square root of a negative number (unless you use imaginary numbers, but we aren't doing that here), so we must exclude negative numbers:

The Domain of \( \sqrt{x} \) is all non-negative Real Numbers

On the Number Line it looks like:

Using set-builder notation it is written: \( \{ x \in \mathbb{R} \mid x \geq 0 \} \)

Or using interval notation it is: \( [0, +\infty) \)

It is important to get the Domain right, or you will get bad results!

So how do you work out the new domain after doing an operation?

**How to Work Out the New Domain**

When you do operations on functions, you end up with the restrictions of both.

It is like cooking for friends:

- one can't eat peanuts,
- the other can't eat dairy food.

So what you cook can't have peanuts and also can't have dairy products.

Example: \( f(x) = \sqrt{x} \) and \( g(x) = \sqrt{3 - x} \)

The domain for \( f(x) = \sqrt{x} \) is from 0 onwards:

The domain for \( g(x) = \sqrt{3 - x} \) is up to and including 3:

The new domain (after adding or whatever) is therefore from 0 to 3:

If you choose any other value, then one or the other part of the new function won't work. In other words you want to find where the two domains **intersect**.
Algebra 1

Note: we can put this whole idea into one line using Set Builder Notation:

\[ \text{Dom}(f+g) = \{ x \in \mathbb{R} \mid x \in \text{Dom}(f) \text{ and } x \in \text{Dom}(g) \} \]

Which says "the domain of f plus g is the set of all Real Numbers that are in the domain of f AND in the domain of g".

The same rule applies when you add, subtract, multiply or divide, except divide has one extra rule.

**An Extra Rule for Division**

There is an *extra rule* for division: As well as restricting the domain as above, when we *divide*:

\[ (f/g)(x) = \frac{f(x)}{g(x)} \]

we must also make sure that \( g(x) \) is *not equal to zero* (so we don't divide by zero).

**Example:** \( f(x) = \sqrt{x} \) and \( g(x) = \sqrt{3-x} \)

\[ (f/g)(x) = \frac{\sqrt{x}}{\sqrt{3-x}} \]

The domain for \( f(x) = \sqrt{x} \) is from 0 onwards:

The domain for \( g(x) = \sqrt{3-x} \) is up to and including 3:

But we also have the restriction that \( \sqrt{3-x} \) *cannot be zero*, so \( x \) cannot be 3:

(Notice the *open circle* at 3, which means not including 3)

So all together we end up with:

**Summary**

- To add, subtract, multiply or divide functions just do as the operation says.
- The domain of the new function will have the restrictions of both functions that made it.
- Divide has the extra rule that the function you are dividing by cannot be zero.
Piecewise Functions

A Function Can be in Pieces

You can create functions that behave differently depending on the input (x) value.

Example: A function with three pieces:

- when x is less than 2, it gives \( x^2 \),
- when x is exactly 2 it gives 6
- when x is more than 2 and less than or equal to 6 it gives the line \( 10-x \)

It looks like this:

And this is how you write it:

\[
f(x) = \begin{cases} 
  x^2 & \text{if } x < 2 \\
  6 & \text{if } x = 2 \\
  10 - x & \text{if } x > 2 \text{ and } x \leq 6 
\end{cases}
\]

The Domain is all Real Numbers up to and including 6:

\[\text{Dom}(f) = (-\infty, 6] \text{ (using Interval Notation)}\]

\[\text{Dom}(f) = \{x \in \mathbb{R} | x \leq 6\} \text{ (using Set Builder Notation)}\]
And here are some example values:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>16</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Example: Here is another piecewise function:

\[ h(x) = \begin{cases} 
2, & \text{if } x \leq 1 \\
 x, & \text{if } x > 1 
\end{cases} \]

which looks like:

The Absolute Value Function

The Absolute Value Function is a famous Piecewise Function. It has two pieces:

- below zero: \(-x\)
- from 0 onwards: \(x\)

\[ f(x) = |x| = \begin{cases} 
x, & \text{if } x \geq 0 \\
-x, & \text{if } x < 0 
\end{cases} \]

The Floor Function

The Floor Function is a very special piecewise function. It has an infinite number of pieces:
Transformations

Just like Transformations in Geometry, you can move and resize the graphs of functions.

Let us start with a function, in this case it is \( f(x) = x^2 \), but it could be anything:

\[ f(x) = x^2 \]

Here are some simple things you can do to move or scale it on the graph:

**You can move it up or down by adding a constant to the y-value:**

\[ g(x) = x^2 + C \]

Note: if you want to move the line down, just use a negative value for C.

- \( C > 0 \) moves it up
- \( C < 0 \) moves it down

**You can move it left or right by adding a constant to the x-value:**

\[ g(x) = (x+C)^2 \]

Adding \( C \) moves the function to the left (the negative direction).

**Why?** Well imagine you are going to inherit a fortune when your age = 25. If you change that to \( (\text{age}+4) = 25 \) then you would get it when you are 21. Adding 4 made it happen earlier.

- \( C > 0 \) moves it left
- \( C < 0 \) moves it right
An easy way to remember what happens to the graph when you add a constant:

- **add to y**: go **high**
- **add to x**: go **left**

**BUT** you must **add C wherever x appears** in the function (you are substituting x+C for x).

**Example: the function** \( v(x) = x^3 - x^2 + 4x \)

Move C spaces to the left: \( w(x) = (x+C)^3 - (x+C)^2 + 4(x+C) \)

You can stretch or compress it in the y-direction by multiplying the whole function by a constant.

\[
g(x) = 0.35(x^2)
\]

- C > 1 stretches it
- 0 < C < 1 compresses it

You can stretch or compress it in the x-direction by multiplying x (wherever it appears) by a constant.

\[
g(x) = (2x)^2
\]

- C > 1 compresses it
- 0 < C < 1 stretches it

Note that (unlike for the y-direction), **bigger** values cause more **compression**.
Algebra 1

You can flip it upside down by multiplying the whole function by -1:

\[ g(x) = -(x^2) \]

This is also called \textbf{reflection about the x-axis} (the axis where \( y=0 \))

You can combine a negative value with a scaling.

Example: multiplying by -2 will flip it upside down AND stretch it in the y-direction.

You can flip it left-right by multiplying the x-value by -1:

\[ g(x) = (-x)^2 \]

It really does flip it left and right! But you can't see it, because \( x^2 \) is symmetrical about the y-axis. So here is another example using \( \sqrt{-x} \):

\[ g(x) = \sqrt{-x} \]

This is also called \textbf{reflection about the y-axis} (the axis where x=0)

\textbf{Summary}

| \( y = f(x) + C \) | \begin{itemize} \item C > 0 moves it up \item C < 0 moves it down \end{itemize} | \( y = f(Cx) \) | \begin{itemize} \item C > 1 compresses it in the x-direction \item 0 < C < 1 stretches it \end{itemize} |
| \( y = f(x + C) \) | \begin{itemize} \item C > 0 moves it left \item C < 0 moves it right \end{itemize} | \( y = -f(x) \) | \textbf{Reflects it about x-axis} |
| \( y = C f(x) \) | \begin{itemize} \item C > 1 stretches it in the y-direction \item 0 < C < 1 compresses it \end{itemize} | \( y = f(-x) \) | \textbf{Reflects it about y-axis} |
You can do all transformation in one go using this:

\[ a f(b(x + c)) + d \]

a is vertical stretch/compression
- \[|a| > 1\] stretches
- \[|a| < 1\] compresses
- \[a < 0\] flips the graph upside down

b is horizontal stretch/compression
- \[|b| > 1\] compresses
- \[|b| < 1\] stretches
- \[b < 0\] flips the graph left-right

c is horizontal shift
- \[c < 0\] shifts to the right
- \[c > 0\] shifts to the left

d is vertical shift
- \[d > 0\] shifts upward
- \[d < 0\] shifts downward

Example: \(2\sqrt{x+1}+1\)

\[a=2, \ c=1, \ d=1\]

So it takes the square root function, and then
- Stretches it by 2 in the y-direction
- Shifts it left 1, and
- Shifts it up 1
**Inverse Functions**

An inverse function goes in the opposite direction!

Let us start with an example:

Here we have the function $f(x) = 2x+3$, written as a flow diagram:

The Inverse Function just goes the other way:

So the inverse of $2x+3$ is: $(y-3)/2$

The inverse is usually shown by putting a little "-1" after the function name, like this:

$f^{-1}(y) ; \text{We say } "f \text{ inverse of } y"$

So, the inverse of $f(x) = 2x+3$ is written: $f^{-1}(y) = (y-3)/2$

(I also used $y$ instead of $x$ to show that we are using a different value.)

**Back to Where We Started**

The cool thing about the inverse is that it should give you back the original value:

If the function $f$ turns the apple into a banana, Then the inverse function $f^{-1}$ turns the banana back to the apple
Example:

Using the formulas from above, we can start with x=4:

\[ f(4) = 2\times4 + 3 = 11 \]

We can then use the inverse on the 11:

\[ f^{-1}(11) = (11-3)/2 = 4 \]

And we magically get 4 back again!

We can write that in one line:

\[ f^{-1}( f(4) ) = 4 \]

"f inverse of f of 4 equals 4"

So applying a function f and then its inverse \( f^{-1} \) gives us the original value back again:

\[ f^{-1}( f(x) ) = x \]

We could also have put the functions in the other order and it still works:

\[ f( f^{-1}(x) ) = x \]

Example:

Start with:

\[ f^{-1}(11) = (11-3)/2 = 4 \]

And then:

\[ f(4) = 2\times4 + 3 = 11 \]

So we can say:

\[ f( f^{-1}(11) ) = 11 \]

"f of f inverse of 11 equals 11"
Algebra 1

Solve Using Algebra

You can work out the inverse using Algebra. Put "y" for "f(x)" and solve for x:

The function: $f(x) = 2x + 3$
Put "y" for "f(x)":
$y = 2x + 3$
Subtract 3 from both sides:
$y - 3 = 2x$
Divide both sides by 2:
$(y - 3)/2 = x$
Swap sides:
$x = (y - 3)/2$
Solution (put "$f^{\text{-1}}(y)$" for "x"):
$f^{\text{-1}}(y) = (y - 3)/2$

This method works well for more difficult inverses.

Fahrenheit to Celsius

A useful example is converting between Fahrenheit and Celsius:

To convert Fahrenheit to Celsius: $f(F) = (F - 32) \times \frac{5}{9}$
The Inverse Function (Celsius back to Fahrenheit) is: $f^{\text{-1}}(C) = (C \times \frac{9}{5}) + 32$

For You: see if you can do the steps to create that inverse!

Inverses of Common Functions

<table>
<thead>
<tr>
<th>Inverses</th>
<th>Careful!</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\times$</td>
<td>$\div$</td>
</tr>
<tr>
<td>$\frac{1}{x}$</td>
<td>$\frac{1}{y}$</td>
</tr>
<tr>
<td>$x^2$</td>
<td>$\sqrt{y}$</td>
</tr>
<tr>
<td>$x^n$</td>
<td>$\sqrt[n]{y}$ or $y^{\frac{1}{n}}$</td>
</tr>
</tbody>
</table>

Don't divide by zero
x and y not zero
x and y ≥ 0
n not zero
(different rules when n is odd, even, negative or positive)
Algebra 1

Did you see the "Careful!" column above? That is because some inverses work **only with certain values**.

**Example: Square and Square Root**

If you square a **negative** number, and then do the inverse this happens:

Square: \((-2)^2 = 4\)
Inverse (Square Root): \(\sqrt{4} = 2\)

But we didn't get the original value back! We got 2 instead of -2. Our fault for not being careful!

So the square function (as it stands) **does not have an inverse**

But we can fix that!

**Restrict the Domain** (the values that can go into a function).

**Example: (continued)**

Just make sure you don't use negative numbers.

In other words, restrict it to \(x \geq 0\) and then you can have an inverse.

So we have this situation:

- \(x^2\) **does not** have an inverse
- but \(\{x^2 \mid x \geq 0\}\) (which says "x squared such that x is greater than or equal to zero" using set-builder notation) **does** have an inverse.

**Why Would There Be No Inverse?**

Let us see graphically what is going on here:

To be able to have an inverse you need **unique values**.

Just think ... if there are two or more **x-values** for one **y-value**, how do you know which one to choose when going back?.
When a y-value has more than one x-value, how do you know which x-value to go back to?

So we have this idea of "a unique y-value for every x-value", and it actually has a name. It is called "Injective" or "One-to-one": If a function is "One-to-one" (Injective) it has an inverse.

Domain and Range

As it stands the function above does not have an inverse.

But you could restrict the domain so there is a unique y for every x ...

... and now you can have an inverse:

Note also:

- The function \( f(x) \) goes from the domain to the range,
- The inverse function \( f^{-1}(y) \) goes from the range back to the domain.

Or...You could plot them both in terms of x ... so it is now \( f^{-1}(x) \), not \( f^{-1}(y) \).

\( f(x) \) and \( f^{-1}(x) \) are like mirror images (flipped about the diagonal).

In other words:

1. The graph of \( f(x) \) and \( f^{-1}(x) \) are symmetric across the line \( y=x \)
Example: Square and Square Root (continued)

First, we restrict the Domain to \( x \geq 0 \):

- \( \{x^2 \mid x \geq 0\} \) "\( x \) squared such that \( x \) is greater than or equal to zero"
- \( \{\sqrt{x} \mid x \geq 0\} \) "square root of \( x \) such that \( x \) is greater than or equal to zero"

And you can see they are "mirror images" of each other about the diagonal \( y=x \).

Note: we could have restricted the domain to \( x \leq 0 \) and the inverse would then be \( f^{-1}(x) = -\sqrt{x} \):

- \( \{x^2 \mid x \leq 0\} \)
- \( \{-\sqrt{x} \mid x \geq 0\} \)

Which are inverses, too.

Not Always Solvable!

It is sometimes not possible to find an Inverse of a Function.

Notes on Notation

Even though we write \( f^{-1}(x) \), the "\(-1\)" is not an exponent (or power):

- \( f^{-1}(x) \) is different to...
  - \( f(x)^{-1} = 1/f(x) \) (the Reciprocal)

Summary

- The inverse of \( f(x) \) is \( f^{-1}(y) \)
- You can find an inverse by reversing the "flow diagram"
- Or you can find an inverse by using Algebra:
  - Put "\( y \)" for "\( f(x) \)", and
  - Solve for \( x \)
- You may need to restrict the domain for the function to have an inverse
Algebra 1 Semester 1 Self-Test

1) Which of the following is a set of Integers.
   A. \(\{1, 2, 3, \ldots\}\)
   B. \(\{0, 1, 2, 3, \ldots\}\)
   C. \(\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}\)
   D. \(\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}\)

2) What property is being exemplified: \((a \times b) \times c = a \times (b \times c)\)
   A. commutative property of addition
   B. commutative property of multiplication
   C. associative property of addition
   D. associative property of multiplication

3) Which of the following is equivalent to \(4(x + 5) - 3(x + 2) = 14\)
   A. \(4x + 20 - 3x - 6 = 14\)
   B. \(4x + 5 - 3x + 6 = 14\)
   C. \(4x + 5 - 3x + 2 = 14\)
   D. \(4x + 20 - 3x - 2 = 14\)

4) Simplify: \(\frac{x^5y^2z}{x^3y^6}\)
   A. \(x^8y^6z\)
   B. \(\frac{y^4z}{x^2}\)
   C. \(\frac{x^2z}{y^4}\)
   D. \(\frac{y^4}{x^2z}\)

5) Simplify: \((x^2y^3)^2 \times (x^3y)^3\)
   A. \(x^{24}y^{18}\)
   B. \(x^{13}y^9\)
   C. \(x^{10}y^9\)
   D. \(x^{36}y^{24}\)

6) 1.256097 rounded to the nearest thousandth is:
   A. 1.256
   B. 1.26
   C. 1.25
   D. 1.2561

7) Change the fraction \(\frac{36}{48}\) into a percentage.
   A. 65%
   B. 75%
   C. 80%
   D. 85%
8) What is the first step in evaluating \(6[2(6 - 4)]\) \(\div 4\)?
   A. \(6 - 4\)
   B. \(2 \times 6\)
   C. \(2 \div 4\)
   D. \(6(6)\)

9) If a can of soup contains 22.0 oz (ounces) of soup, how many grams of soup is that?
   Round answer to the nearest whole place. \((1 \text{ lb} = 16 \text{ oz}, 1 \text{ lb} = 454 \text{ g})\)
   A. 20.6 g
   B. 330 g
   C. 523 g
   D. 624 g

10) Solve: \((x - 4)(x + 7) = 0\)
    A. \(x = 4 \text{ or } x = 7\)
    B. \(x = -4 \text{ or } x = -7\)
    C. \(x = 4 \text{ or } x = -7\)
    D. \(x = -4 \text{ or } x = 7\)

11) Use the standard form of the zero product property to solve \(3(x - 2) = 3x(x - 2)\)
    A. The solution is -1 only
    B. The solutions are -1 and -2
    C. The solution is 1 only
    D. The solutions are 1 and 2

12) Which ordered pair is the solution of the equation \(5x + y = -23\)?
    A. (-5, 2)
    B. (2, -5)
    C. (1, -5)
    D. (-5, 1)

13) Is the relation depicted in the chart below a function?

\[
\begin{array}{ccccccc}
 x & -2 & 3 & -6 & 4 & -7 & 5 \\
 y & 3 & -6 & 3 & -6 & 3 & -6 \\
\end{array}
\]

   A. Yes
   B. No
   C. Cannot be determined

14) Which graph represents a function?
15) Find \((f \circ g)(-4)\) when \(f(x) = 4x + 5\) and \(g(x) = 4x^2 - 5x - 3\)
 A. 9 
 B. 536 
 C. 329 
 D. 8 

16) Give the domain and range of the relation.

\[\begin{array}{c}
\text{A. } D: 0 \leq x \leq 7; R: 1 \leq y \leq 7 \\
\text{B. } D: 1 \leq x \leq 6; R: 1 \leq y \leq 7 \\
\text{C. } D: 1 \leq x \leq 7; R: 1 \leq y \leq 6 \\
\text{D. } D: 1 \leq x \leq 7; R: 1 \leq y \leq 6 \\
\end{array}\]

17) For \(f(x) = \frac{x}{x+1}\) and \((g) = \frac{1}{x-2}\), find the Domain of \((f \circ g)(x)\).
 A. \(x \neq -1\) and \(x \neq 2\) 
 B. \(x \neq -1\) and \(x \neq 2\) 
 C. \(x \neq 1\) and \(x \neq 2\) 
 D. The domain is all real numbers 

18) Determine which the graph of the given function is. \(f(x)\) \[
\begin{cases}
3x + 2 & \text{if } x < 5 \\
-3x - 2 & \text{if } x \geq 5 
\end{cases}\]

\[\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.} \\
\text{d.} \\
\end{array}\]
19) Begin by graphing the standard absolute value function \( f(x) = |x| \). Then use transformations of this graph to graph the given function.

\[ g(x) = |x| + 3 \]

![Graphs](image)

A.  
B.  
C.  
D.  

20) Which pair of graphs shows an exponential function and its inverse?

![Graphs](image)

A.  
B.  
C.  
D.  

21) Which of the following is the inverse to the function \( f(x) = -8 - 5x \)?

A.  \( f^{-1}(x) = \frac{1}{5x} - \frac{1}{8} \)
B.  \( f^{-1}(x) = -\frac{x}{5} + \frac{8}{5} \)
C.  \( f^{-1}(x) = -\frac{x}{5} - \frac{8}{5} \)
D.  \( f^{-1}(x) = 5x + 8 \)