Geometry

Semester 1
Common Core State Standards Initiative

COURSE DESCRIPTION

Geometry Semester 1

Will meet graduation requirements for
Geometry Semester 1

Subject Area: Mathematics

Course Number: 1206310

Course Title: Geometry Semester 1

Credit: 0.5
Introduction

American Worldwide Academy’s math course, AWA Geometry, focuses on the fundamental skills that are necessary for understanding the basics of geometry. This Study guide addresses terminology and fundamental properties as well as introducing reasoning. AWA Geometry is full of practical, useful information geared to helping students recover credit for geometry while mastering the basics. This Study guide will be helpful to any student who has previously had difficulties with understanding geometric concepts and skills.

There are two sections that cover core topics of geometry at the first course level. At the beginning of each section of study, you will see the objectives outlined that will help you master the standards for the section.
Course Objectives
After successful completion of this course, students will know and be able to do the following:

Geometry Standards and Concepts

Section 1: Points, Lines, Angles, and Planes-
Demonstrate an understanding of the terminology and fundamental properties of geometry.

- Understand geometric concepts such as perpendicularity, parallelism, tangency, congruency, similarity, reflections, symmetry, and transformations including flips, slides, turns, enlargements, rotations, and fractals.
- Analyze and apply geometric relationships involving planar cross-sections (the intersection of a plane and a three dimensional figure).

Section 2: Deductive and Inductive Reasoning –
Demonstrate an understanding of deductive and inductive reasoning.

- Use properties and relationships of geometric shapes to construct formal and informal proofs.
Geometry

❖ Getting Started

You will learn much from this course that will help you in your future studies and career. In addition to reviewing and completing the study guide and textbook, your Final Examination will be evidence that you have mastered the standards for geometry. You will know the concepts and be able to do the skills that will earn you one full credit for Geometry.

If you are ready to begin, turn to the next page in this Study guide: the Progress Chart and Self-Test Schedule, which will serve as a guide to help you move through the course. Let’s get started on earning that algebra credit—good luck!
<table>
<thead>
<tr>
<th>Complete and Review:</th>
<th>Record Your Progress:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point, Lines, and Planes</strong></td>
<td></td>
</tr>
<tr>
<td>Undefined Terms</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Intuitive Concepts</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Common Symbols</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td><strong>Postulates and Theorem</strong></td>
<td></td>
</tr>
<tr>
<td>Building Blocks of Proof</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Postulates and Axioms</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Theorems</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td><strong>An Angle in Geometry</strong></td>
<td></td>
</tr>
<tr>
<td>Type of Angles</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Rules</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td><strong>Parallel and Perpendicular Lines</strong></td>
<td></td>
</tr>
<tr>
<td>Parallel Lines</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Perpendicular Lines</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Corresponding Angles</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Parallelogram</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Rectangle</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Square</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td><strong>Transformations</strong></td>
<td></td>
</tr>
<tr>
<td>Translation</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Rotation</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Reflection</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Glide Reflection</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td><strong>Similar Polygons</strong></td>
<td></td>
</tr>
<tr>
<td>Similar Quadrilaterals</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Similar Triangles</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td><strong>Deductive and Inductive Reasoning</strong></td>
<td></td>
</tr>
<tr>
<td>Terms</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Inductive Reasoning</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Deductive Reasoning</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td><strong>Converse, Inverse, Contrapositive</strong></td>
<td></td>
</tr>
<tr>
<td>Converse</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Inverse</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
<tr>
<td>Contrapositive</td>
<td>Date completed: <strong><strong>/__/</strong></strong></td>
</tr>
</tbody>
</table>
# Geometry

## Table of Contents

### Section 1: Terminology and Fundamental Properties of Geometry

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points, Lines, and Planes</td>
<td>8</td>
</tr>
<tr>
<td>Postulates and Theorems</td>
<td>10</td>
</tr>
<tr>
<td>Angle in Geometry</td>
<td>13</td>
</tr>
<tr>
<td>Parallel and Perpendicular Lines</td>
<td>14</td>
</tr>
<tr>
<td>Transformations</td>
<td>17</td>
</tr>
<tr>
<td>Similar Polygons</td>
<td>18</td>
</tr>
</tbody>
</table>

### Section 2: Deductive and Inductive Reasoning

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms</td>
<td>22</td>
</tr>
<tr>
<td>Inductive Reasoning</td>
<td>23</td>
</tr>
<tr>
<td>Deductive Reasoning</td>
<td>24</td>
</tr>
<tr>
<td>Converse, Inverse, Contrapositive</td>
<td>25</td>
</tr>
</tbody>
</table>
Geometry

Content Review for Section 1: Terminology and Fundamental Properties of Geometry

Points, Lines, and Planes

Undefined terms

In geometry, definitions are formed using known words or terms to describe a new word. There are three words in geometry that are not formally defined. These three undefined terms are point, line and plane.

**POINT** - a point has no dimension (actual size). Even though we represent a point with a dot, the point has no length, width, or thickness. Our dot can be very tiny or very large and it still represents a point. A point is usually named with a capital letter. In the coordinate plane, a point is named by an ordered pair, \((x,y)\).

**LINE** - In geometry, a line has no thickness but its length extends in one dimension and goes on forever in both directions. Unless otherwise stated a line is drawn as a straight line with two arrowheads indicating that the line extends without end in both directions. A line is named by a single lowercase letter, \(\ell\), or by any two points on the line, \(\overline{AB}\).

**PLANE** - In geometry, a plane has no thickness but extends indefinitely in all directions. Planes are usually represented by a shape that looks like a tabletop or a parallelogram. Even though the diagram of a plane has edges, you must remember that the plane has no boundaries. A plane is named by a single letter (plane \(m\)) or by three non-collinear points (plane ABC).

Intuitive Concepts

There are a few basic concepts in geometry that need to be understood, but are seldom used as reasons in a formal proof.
Geometry

<table>
<thead>
<tr>
<th><strong>Collinear Points</strong></th>
<th>points that lie on the same line.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coplanar points</strong></td>
<td>points that lie in the same plane.</td>
</tr>
<tr>
<td><strong>Opposite rays</strong></td>
<td>2 rays that lie on the same line, with a common endpoint and no other points in common. Opposite rays form a straight line and/or a straight angle (180°).</td>
</tr>
<tr>
<td><strong>Parallel lines</strong></td>
<td>two coplanar lines that do not intersect</td>
</tr>
<tr>
<td><strong>Skew lines</strong></td>
<td>two non-coplanar lines that do not intersect.</td>
</tr>
</tbody>
</table>

**Common Symbols Used in Geometry**

Symbols save time and space when writing. Here are the most common geometrical symbols:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Example</th>
<th>In Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ</td>
<td>Triangle</td>
<td>ΔABC has 3 equal sides</td>
<td>Triangle ABC has three equal sides</td>
</tr>
<tr>
<td>∠</td>
<td>Angle</td>
<td>∠ABC is 45°</td>
<td>The angle formed by ABC is 45 degrees.</td>
</tr>
<tr>
<td>⊥</td>
<td>Perpendicular</td>
<td>AB ⊥ CD</td>
<td>The line AB is perpendicular to line CD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Parallel</td>
</tr>
<tr>
<td>°</td>
<td>Degrees</td>
<td>360° makes a full circle</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Right Angle (90°)</td>
<td>L is 90°</td>
<td>A right angle is 90 degrees</td>
</tr>
<tr>
<td>(\overline{AB})</td>
<td>Line Segment &quot;AB&quot;</td>
<td>AB</td>
<td>The line between A and B</td>
</tr>
<tr>
<td>(\overleftrightarrow{AB})</td>
<td>Line &quot;AB&quot;</td>
<td>(\overleftrightarrow{AB})</td>
<td>The infinite line that includes A and B</td>
</tr>
<tr>
<td>(\overrightarrow{AB})</td>
<td>Ray &quot;AB&quot;</td>
<td>(\overrightarrow{AB})</td>
<td>The line that starts at A, goes through B and continues on</td>
</tr>
<tr>
<td>≅</td>
<td>Congruent (same shape and size)</td>
<td>ΔABC ≅ ΔDEF</td>
<td>Triangle ABC is congruent to triangle DEF</td>
</tr>
<tr>
<td>~</td>
<td>Similar (same shape, different size)</td>
<td>ΔDEF ~ ΔMNO</td>
<td>Triangle DEF is similar to triangle MNO</td>
</tr>
<tr>
<td>. . .</td>
<td>Therefore</td>
<td>a = b . . . b = a</td>
<td>a equals b, therefore b equals a</td>
</tr>
</tbody>
</table>

Example:
When someone writes: In ΔABC, ∠BAC is L.
They are really saying: "In triangle ABC, the angle BAC is a right angle"
Geometry

Building Blocks of Proof

Just as figures in a plane are made of building blocks such as points, segments, and lines, geometric proofs are made of building blocks, too. These building blocks include definitions, postulates, axioms, and theorems. Together, these building blocks are combined to make each step of a proof.

In the previous section of this study guide, we learned the definitions of many terms. These definitions are used all the time in geometric proofs. For example, as we will learn, a polygon with three sides is a triangle. This is a definition. In a proof, we might be confronted with a three-sided polygon, but not know much about it. Using the definition of a triangle, we could deduce that the polygon is a triangle, and with that knowledge, and our knowledge of triangles, we could deduce much about the previously unknown polygon. In this way, definitions are used in proofs.

Whenever a number of terms are defined, there must be a foundation of terms that are understood to begin with. Without the use of such terms, every definition would be circular—words would be defined in terms of themselves. These terms in geometry are called undefined terms. The undefined terms are the building blocks of geometric figures, like points, lines, and planes. Points, lines, and planes don't have specific, universal definitions. They are explained in any text as clearly as possible, so that every following term can be explained using a combination of undefined terms as well as previously defined terms. Only with an understanding of these undefined terms can other terms, like polygons, for example, be defined.

Postulates and Axioms

Postulates and axioms are statements that we accept as true without proof. They are essentially the same thing, and in many textbooks, a distinction isn't made between them. But more often than not, axioms are statements or properties of real numbers, whereas postulates are statements or properties of geometric figures.

Although axioms and postulates pertain to different concepts (numbers, and figures, that is), they play the same role in geometric proofs: they form the foundation of theorems. The reason we accept both without proof is that postulates and axioms often can't be proved. Take the parallel postulate, for example. It states that given a line and a point not on that line, exactly one line can be drawn that contains the point and is parallel to the line. There is no formal proof for this, but it is doubtlessly true. In this way, Postulates and axioms are much like undefined terms. Just as undefined terms are accepted without formal definitions (the concepts are explained, but not really defined), and a foundation of undefined terms makes it possible to define a wide range of other terms, postulates and axioms are understood to be
true even though we have no formal way to prove their truth. With a foundation of a few postulates or axioms, countless theorems can be proved.

**Theorems**

Theorems are statements that can be proved. Once a theorem has been proved, it can be used in other proofs. This way the bank of geometric knowledge builds up so that one doesn't need to re-prove the simplest properties. For example, the fact that the diagonals of a rectangle are congruent is a very basic theorem that can be proved using the SAS method of proving triangles congruent. (The method for formally proving this will come later.) In a future proof, you can simply state that the diagonals of a rectangle are congruent according to this theorem instead of having to go through the SAS method again.

Proving the truth of certain universal geometric properties is one of the ultimate goals of geometry. Most of the things we'll prove will be congruent in specific situations; to prove the general fact that the diagonals of any rectangle are congruent is a much greater accomplishment.

**Problems**

**Problem:** What is the difference between an axiom and a postulate?

Solution for Problem 1 >> An axiom governs real numbers, whereas a postulate governs geometric figures.

**Problem:** What are the valid reasons for drawing conclusions in a proof (the building blocks of proof?)

Solution for Problem 2 >> Definitions, axioms, postulates, and theorems

A postulate is a statement that is assumed true without proof. A theorem is a true statement that can be proven. Listed below are six postulates and the theorems that can be proven from these postulates

- **Postulate 1:** A line contains at least two points.
- **Postulate 2:** A plane contains at least three noncollinear points.
- **Postulate 3:** Through any two points, there is exactly one line.
- **Postulate 4:** Through any three noncollinear points, there is exactly one plane.
- **Postulate 5:** If two points lie in a plane, then the line joining them lies in that plane.
- **Postulate 6:** If two planes intersect, then their intersection is a line.
- **Theorem 1:** If two lines intersect, then they intersect in exactly one point.
- **Theorem 2:** If a point lies outside a line, then exactly one plane contains both the line and the point.
- **Theorem 3:** If two lines intersect, then exactly one plane contains both lines.
Example 1: State the postulate or theorem you would use to justify the statement made about each figure.

- (a) Through any three noncollinear points, there is exactly one plane (*Postulate 4*).
- (b) Through any two points, there is exactly one line (*Postulate 3*).
- (c) If two points lie in a plane, then the line joining them lies in that plane (*Postulate 5*).
- (d) If two planes intersect, then their intersection is a line (*Postulate 6*).
- (e) A line contains at least two points (*Postulate 1*).
- (f) If two lines intersect, then exactly one plane contains both lines (*Theorem 3*).
- (g) If a point lies outside a line, then exactly one plane contains both the line and the point (*Theorem 2*).
- (h) If two lines intersect, then they intersect in exactly one point (*Theorem 1*).
Geometry

❖ An Angle in Geometry

An angle is the rotation required to superimpose one of two intersecting lines on the other.

A Right Angle

A right angle is an angle with measure equal to 90 degrees.

An Acute Angle

An acute angle is an angle with a measure between 0 and 90 degrees.

An Obtuse Angle

An obtuse angle is an angle with a measure between 90 and 180 degrees.

Adjacent angles

Adjacent angles are any two angles that share a common side separating the two angles and that share a common vertex.

Vertical angles

Vertical angles are formed when two lines intersect and form four angles. Any two of these angles that are not adjacent angles are called vertical angles. Vertical angles are always congruent.

Example: angle 1 ≅ 3, and likewise, angle 4 ≅ 2.

Complementary Angles

Two angles are complementary if the sum of their measures is equal to 90 degrees.

Example: angles a and b with measures a = 19° and b = 71° are complementary since a + b = 90°

Supplementary Angles

Two angles are supplementary if the sum of their measures is equal to 180 degrees.

Example: angles a and b with measures a = 122.1° and b = 57.9° are supplementary since a + b = 180°
Parallel and Perpendicular Lines

Two lines or line segments can either intersect (cross) each other or be parallel. Think of the parallel lines as never meeting each other, no matter how much you would continue them to both directions.

These lines intersect.

These lines are parallel.

What about these lines? Do they intersect or are they parallel? Try continuing the lines with your ruler and see what happens.

These lines intersect and form four right angles. They are perpendicular lines.

The little symbol ("corner") is used to indicate a right angle.

We already know that two lines that intersect form two pairs of vertical angles. What happens if you have three lines, and two of them are parallel?

How many angles are there in the picture?

Measure the angles. Which ones have the same measure?

Again you see two parallel lines and one line that intersects them both. The two angles marked with a single arc are corresponding angles. Also the two angles marked with the double arc are corresponding angles. All four angles have the same measure.
What kind of lines do you see in this picture?

How many angles?

Which angles have the same measure?

Mark the rest of the angles in this picture and measure them.

The figure that is enclosed by the lines is called a parallelogram.

A figure that has four sides so that the two opposite sides are parallel and the other two sides are parallel to each other, is called a parallelogram.

What angles in the parallelogram have the same measure?

If all the angles in a parallelogram are right angles, it is called a rectangle.

If all the four angles are right angles and all the four sides are the same, the figure is called a square.
Geometry

Exercises

(see solutions below) Given angles a, b, c, d, e and f with measures

\[ a = 21^\circ, \ b = 90.1^\circ, \ c = 90^\circ, \ d = 134.2^\circ, \ e = 69^\circ, \ f = 45.8^\circ \]

1 - Which of these angles are acute? \ 1 - a, e and f

2 - Which of these angles are obtuse? \ 2 - b and d.

3 - Which pairs of angles are complementary? \ 3 - a - e

4 - Which pairs of angles are supplementary? \ 4 - d - f

Example problem types

1. Calculate the other angles inside the parallelogram. Do not measure since the pictures are not exact. What are the principles you can use?

![Parallelogram diagram]

2. a) Draw a parallelogram that has a 50° angle. You can decide the length of the sides of the parallelogram. What else do you need to know to be able to draw the parallelogram?

   b) Draw a parallelogram that has a 118 angle.

5. Find one corresponding angle to each of the marked angles and mark them the same way. These pictures do have some extra lines there too but remember you need to look for two parallel lines and a line that intersects them both.
Geometry

### Transformations

A **transformation** is a mapping $f$ of $A$ onto $B$ such that each elements of $B$ is the image of exactly one element of $A$.

A **transformation $f$** is called an **isometry** of $A$ onto $B$ if it preserves distances.

**NOTE:** The four basic Euclidean transformations: rotation, translations, reflection and glide reflections, are all isometries.

### Types of Euclidean Transformations

1. A **translation** is a correspondence between points and their image points so that each image is the same distance in the same direction from the original point.

2. A **rotation** is a correspondence between points and their image points where one point is fixed and the image points are transformed at a new angle position. The example below shows 5 rotations of the original shape around the center point:

3. A **reflection** is a correspondence between points and their image points so that each image is transformed as a mirror image over a horizontal(vertical or other) line.

4. A **glide reflection** is a correspondence between points and their image points where the image points are the product of a reflection and a translation parallel to the fixed line of reflection. This is often used in ornamental patterns - seen especially in the Alhambra in Grenada, Spain.
Similar Polygons

Similar Quadrilaterals

Two polygons with the same shape are called similar polygons. The symbol for “is similar to” is \( \sim \). Notice that it is a portion of the “is congruent to” symbol, \( \cong \). When two polygons are similar, these two facts both must be true:

- Corresponding angles are equal.
- The ratios of pairs of corresponding sides must all be equal.

In Figure 1, quadrilateral \( ABCD \sim \) quadrilateral \( EFGH \).

This means: \( m \angle A = m \angle E \), \( m \angle B = m \angle F \), \( m \angle C = m \angle G \), \( m \angle D = m \angle H \), and

\[
\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{FH}
\]

It is possible for a polygon to have one of the above facts true without having the other fact true. The following two examples show how that is possible. In Figure 2, quadrilateral \( QRST \) is not similar to quadrilateral \( WXYZ \).

Even though the ratios of corresponding sides are equal, corresponding angles are not equal (\( 90^\circ \neq 120^\circ \), \( 90^\circ \neq 60^\circ \)). In Figure 3, quadrilateral \( FGHI \) is not similar to quadrilateral \( JKLM \).

Even though corresponding angles are equal, the ratios of each pair of corresponding sides are not equal (\( \frac{3}{3} \neq \frac{5}{3} \)).
Example 1: In Figure 4, quadrilateral $ABCD \sim$ quadrilateral $EFGH$. (a) Find $m \angle E$. (b) Find $x$.

(a) $m \angle E = 90^\circ$ ($\angle E$ and $\angle A$ are corresponding angles of similar polygons, and corresponding angles of similar polygons are equal.)

(b) $9/6 = 12/ x$ (If two polygons are similar, then the ratios of each pair of corresponding sides are equal.)

$$9x = 72 \text{ (Cross-Products Property)}$$

$$x = 8$$

Similar Triangles

In general, to prove that two polygons are similar, you must show that all pairs of corresponding angles are equal and that all ratios of pairs of corresponding sides are equal. In triangles, though, this is not necessary.

Postulate 17 (AA Similarity Postulate): If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

Example 1: Use Figure 1 to show that the triangles are similar.

In $\triangle ABC$,

$$m \angle A + m \angle B + m \angle C = 180^\circ$$

$$m \angle A + 100^\circ + 20^\circ = 180^\circ$$

$$m \angle A = 60^\circ$$

But in $\triangle DEF$,

$$m \angle D = 60^\circ$$

So, $m \angle A = m \angle D$

By Postulate 17, the AA Similarity Postulate, $\triangle ABC \sim \triangle DEF$. Additionally, because the triangles are now similar,

$$m \angle C = m \angle F$$

and

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
Geometry

Example 2: Use Figure 2 to show that $\triangle QRS \sim \triangle UTS$.

$m \angle 1 = m \angle 2$, because vertical angles are equal.

$m \angle R = m \angle T$ or $m \angle Q = m \angle U$, because if two parallel lines are cut by a transversal, then the alternate interior angles are equal. So by the AA Similarity Postulate, $\triangle QRS \sim \triangle UTS$.

Example 3: Use Figure 3 to show that $\triangle MNO \sim \triangle PQR$.

In $\triangle MNO$, $MN = NO$, and in $\triangle PQR$, $PQ = QR$.

$m \angle M = m \angle O$ and $m \angle P = m \angle R$

(If two sides of a triangle are equal, the angles opposite these sides have equal measures.)

In $\triangle MNO$, $m \angle M + m \angle N + m \angle O = 180^\circ$

In $\triangle PQR$, $m \angle P + m \angle Q + m \angle R = 180^\circ$

Because $m \angle M = m \angle O$ and $m \angle P = m \angle R$

\[
\begin{align*}
2m \angle M + 40^\circ &= 180^\circ & 2m \angle P + 40^\circ &= 180^\circ \\
2m \angle M &= 140^\circ & 2m \angle P &= 140^\circ \\
m \angle M &= 70^\circ & m \angle P &= 70^\circ
\end{align*}
\]

So, $m \angle M = m \angle P$, and $m \angle O = m \angle R$. $\triangle MNO \sim \triangle PQR$ (AA Similarity Postulate).
Example 4: Use Figure 4 to show that $\triangle ABC \sim \triangle DEF$.

$m \angle C = m \angle F$ (All right angles are equal.)

$m \angle A = m \angle D$ (They are indicated as equal in the figure.)

$\triangle ABC \sim \triangle DEF$ (AA Similarity Postulate)
Geometry

Content Review for Section 2: Deductive and Inductive Reasoning

One of the most important ways to apply that knowledge is through writing geometric proofs. Geometric proofs are ways to logically make an argument about a certain figure or figures in geometrical terms; a formal geometric proof is a meticulous, step-by-step way to show, in most cases, that certain figures or parts thereof are congruent. Using proofs, you could measure one figure or part of a figure and know the measure of another figure or part of that figure which is impossible to measure. Naturally, just as proofs can prove congruence, they can be used to prove incongruence as well.

To learn how to write a formal proof, which is the main purpose of this text, it is necessary to backtrack a little and look at the way a mathematician must reason in order to come to useful conclusions with limited knowledge. Two of the most basic methods of mathematical reasoning are inductive and deductive reasoning. They are both useful ways to arrive at conclusions, and are both very important to the study of geometry. Deductive reasoning is used more heavily than inductive reasoning in geometry, but in all of mathematics, including some of geometry, the process of deductive reasoning is only possible after inductive reasoning has led a mathematician to hypothesize about a given situation: only after a proof has been attempted can a mathematician’s hypothesis be verified or refuted.

In the following lessons, we’ll see exactly how inductive and deductive reasoning are used in geometry. Finally, just as we studied the buildings blocks of geometric figures in Geometry 1, in following lessons we’ll take a look at the building blocks of geometric proofs. These include definitions, postulates, axioms, and theorems. By the end of Geometry 3, we'll fully understand how to use these building blocks to write geometric proofs. The task begins in the next few lessons as we learn about the nature of mathematical reasoning.

Terms

**Axiom** - A statement about real numbers that is accepted as true without proof.
**Deductive Reasoning** - A form of reasoning by which each conclusion follows from the previous one; an argument is built by conclusions that progress towards a final statement.
**Inductive Reasoning** - A form of reasoning in which a conclusion is reached based on a pattern present in numerous observations.
**Postulate** - A statement about geometry that is accepted as true without proof.
**Theorem** - A statement in geometry that has been proved.
**Undefinable Terms** - Terms that aren’t defined, but instead explained; they form the foundation for all definitions in geometry.
Inductive reasoning is the process of arriving at a conclusion based on a set of observations. In itself, it is not a valid method of proof. Just because a person observes a number of situations in which a pattern exists doesn't mean that that pattern is true for all situations. For example, after seeing many people outside walking their dogs, one may observe that every dog that is a poodle is being walked by an elderly person. The person observing this pattern could inductively reason that poodles are owned exclusively by elderly people. This is by no means a method of proof for such a suspicion; in fact, in the real world it is a means by which people and things are stereotyped. A hypothesis based on inductive reasoning, can, however, lead to a more careful study of a situation. By inductive reasoning, in the example above, a viewer has formed a hypothesis that poodles are owned exclusively by elderly people. The observer could then conduct a more formal study based on this hypothesis and conclude that his hypothesis was either right, wrong, or only partially wrong.

Inductive reasoning is used in geometry in a similar way. One might observe that in a few given rectangles, the diagonals are congruent. The observer could inductively reason that in all rectangles, the diagonals are congruent. Although we know this fact to be generally true, the observer hasn’t proved it through his limited observations. However, he could prove his hypothesis using other means (which we'll learn later) and come out with a theorem (a proven statement). In this case, as in many others, inductive reasoning led to a suspicion, or more specifically, a hypothesis, that ended up being true.

The power of inductive reasoning, then, doesn't lie in its ability to prove mathematical statements. In fact, inductive reasoning can never be used to provide proofs. Instead, inductive reasoning is valuable because it allows us to form ideas about groups of things in real life. In geometry, inductive reasoning helps us organize what we observe into succinct geometric hypotheses that we can prove using other, more reliable methods. Whether we know it or not, the process of inductive reasoning almost always is the way we form ideas about things. Once those ideas form, we can systematically determine (using formal proofs) whether our initial ideas were right, wrong, or somewhere in between.

**Problems**

**Problem:** Can inductive reasoning be used to formally prove something?

Solution for Problem 1 >>**No.**

**Problem:** What is the basic role of inductive reasoning in geometry?

Solution for Problem 2 >>**Inductive reasoning leads people to form hypotheses based on observations made. Then these hypotheses can be tested rigorously using other methods. Inductive reasoning is how people make generalizations about sets of things and form hypotheses accordingly.**
Deductive reasoning, unlike inductive reasoning, is a valid form of proof. It is, in fact, the way in which geometric proofs are written. Deductive reasoning is the process by which a person makes conclusions based on previously known facts. An instance of deductive reasoning might go something like this: a person knows that all the men in a certain room are bakers, that all bakers get up early to bake bread in the morning, and that Jim is in that specific room. Knowing these statements to be true, a person could deductively reason that Jim gets up early in the morning. Such a method of reasoning is a step-by-step process of drawing conclusions based on previously known truths. Usually a general statement is made about an entire class of things, and then one specific example is given. If the example fits into the class of things previously mentioned, then deductive reasoning can be used. Deductive reasoning is the method by which conclusions are drawn in geometric proofs.

Deductive reasoning in geometry is much like the situation described above, except it relates to geometric terms. For example, given that a certain quadrilateral is a rectangle, and that all rectangles have equal diagonals, what can you deduce about the diagonals of this specific rectangle? They are equal, of course. An example of deductive reasoning in action.

Although deductive reasoning seems rather simple, it can go wrong in more than one way. When deductive reasoning leads to faulty conclusions, the reason is often that the premises were incorrect. In the example in the previous paragraph, it was logical that the diagonals of the given quadrilateral were equal. What if the quadrilateral wasn’t a rectangle, though? Maybe it was actually a parallelogram, or a rhombus. In such a case, the process of deductive reasoning cannot be used. The fact that the diagonals of a rectangle are equal tells us nothing relevant about the diagonals of a parallelogram or a rhombus. The premises used in deductive reasoning are in many ways the most important part of the entire process of deductive reasoning. If they are incorrect, the foundation of the whole line of reasoning is faulty, and nothing can be reliably concluded. Even if just one conclusion is incorrect, every conclusion after that is unreliable, and may very well be incorrect, also.

Another instance in which deductive reasoning doesn’t work is when it is not executed properly. Using the example in the first paragraph, let’s add the premise that Bob is a baker. Can we deduce that Bob is in the room? We could only deduce this if we knew that everybody who was a baker was in the room. This was not one of the premises, though. When reading premises, it is very important not to assume anything more than exactly what is written. In Logic Statements we’ll more carefully examine exactly what occurs when premises are misused and lead to false conclusions. For now, it is enough to know that deductive reasoning is perfectly effective when all of the premises are true, and each step in the process of deductive reasoning follows logically from the previous step.
**Geometry**

**Problems**

**Problem**: Take the following scenario: Every time a batter reaches first base, the next batter hits a double. Every time a batter hits a double, the runner on first scores. Jon reaches first base. What can you deduce about Jon?

Solution for Problem 1 >> **Jon scores**.

**Problem**: Take the following scenario: When the sun shines, the grass grows. When the grass grows, it needs to be cut. The sun shines. What can you deduce about the grass?

Solution for Problem 2 >> **It needs to be cut**.

**Problem**: Take the following scenario: Jim is a barber. Everybody who gets his hair cut by Jim gets a good haircut. Austin got a good haircut. What can you deduce about Austin?

Solution for Problem 3 >> **Nothing. Just because Austin got a good haircut does not mean that Jim cut his hair. This is always possible, but nothing can be deduced from the situation**.

**Problem**: All dogs are mammals, and all mammals are vertebrates. Shaggy is a dog. What can be deduced about Shaggy?

Solution for Problem 4 >> **Shaggy is a mammal and a vertebrate**.

**Problem**: Why is the following example of deductive reasoning faulty? Given: Khaki pants are comfortable. Comfortable pants are expensive. Adrian's pants are not khaki pants. Deduction: Adrian's pants are not expensive.

Solution for Problem 5 >> **Simply because Adrian's pants are not khaki does not mean that they are not comfortable. Also, even if they are not comfortable, this would not necessarily mean that they are not expensive. There could be comfortable pants that are not khakis, and expensive pants that are not comfortable.**
**Converse, Inverse, Contrapositive**

Given an if-then statement "if $p$, then $q$", we can create three related statements:

A conditional statement consists of two parts, a hypothesis in the “if” clause and a conclusion in the “then” clause. For instance, “If it rains, then they cancel school.”

"It rains" is the hypothesis.
"They cancel school" is the conclusion.

To form the **converse** of the conditional statement, interchange the hypothesis and the conclusion.

The converse of "If it rains, then they cancel school" is "If they cancel school, then it rains."

To form the **inverse** of the conditional statement, take the negation of both the hypothesis and the conclusion.

The inverse of “If it rains, then they cancel school” is “If it does not rain, then they do not cancel school.”

To form the **contrapositive** of the conditional statement, interchange the hypothesis and the conclusion of the inverse statement.

The contrapositive of "If it rains, then they cancel school" is "If they do not cancel school, then it does not rain."

<table>
<thead>
<tr>
<th>Statement</th>
<th>If $p$, then $q$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converse</td>
<td>If $q$, then $p$</td>
</tr>
<tr>
<td>Inverse</td>
<td>If not $p$, then not $q$.</td>
</tr>
<tr>
<td>Contrapositive</td>
<td>If not $q$, then not $p$.</td>
</tr>
</tbody>
</table>

If the statement is true, then the contrapositive is also logically true. If the converse is true, then the inverse is also logically true.

Example 1:

**Statement** If two angles are congruent, then they have the same measure.
**Converse** If two angles have the same measure, then they are congruent.
**Inverse** If two angles are not congruent, then they do not have the same measure.
**Contrapositive** If two angles do not have the same measure, then they are not congruent.

In the above example, since the hypothesis and conclusion are equivalent, all four statements are true. But this will not always be the case!

Example 2:

**Statement** If a quadrilateral is a rectangle, then it has two pairs of parallel sides.
**Converse** If a quadrilateral has two pairs of parallel sides, then it is a rectangle. (FALSE!)
**Inverse** If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides. (FALSE!)
**Contrapositive** If a quadrilateral does not have two pairs of parallel sides, then it is not a rectangle.
Geometry Semester 1 Self-Test

Problems 1-3: Refer to the figure on the right

1.) Name the line that contains point J
   a. \( \overrightarrow{GF} \)
   b. \( \overrightarrow{DB} \)
   c. \( n \)
   d. \( p \)

2.) Name the plane containing lines \( m \) and \( p \)
   a. \( n \)
   b. \( GFC \)
   c. \( H \)
   d. \( JDB \)

3.) What is another name for line \( n \)?
   a. line \( JB \)
   b. \( \overrightarrow{DC} \)
   c. \( \overrightarrow{GF} \)
   d. \( AC \)

4.) How many planes are shown in the figure?
   a. 4
   b. 3
   c. 5
   d. 6

5.) Although the statement “if two points lie in a plane, then the lines containing those points lie in the plane” cannot be proven, it is agreed to be true. Which type of statement is this an example of?
   a. Undefined term
   b. Definition
   c. Postulate (or axiom)
   d. Theorem

6.) Which statement is a theorem?
   a. An endpoint is a point at one end of a segment or the starting point of a ray.
   b. If two angles are complementary to the same angle, then the two angles are congruent.
   c. If two lines intersect, then they intersect in exactly one point.
   d. A plane is a flat surface that has no thickness and extends forever.
Problems 7 & 8: Refer to the figure on the right, in which M, R and Q are collinear and \( m\angle MRN = 90° \):

7.) Which of the following is a straight angle?
   a. \( \angle MRN \)
   b. \( \angle PRN \)
   c. \( \angle NRT \)
   d. \( \angle MRQ \)

8.) Which of the following is an obtuse angle?
   a. \( \angle MRN \)
   b. \( \angle PRN \)
   c. \( \angle NRT \)
   d. \( \angle MRP \)

9.) If \( \angle A \) and \( \angle B \) are complementary, \( \angle B \) and \( \angle C \) are supplementary, and \( m\angle A = 64° \), then what is the measure of \( \angle C \)?
   a. 64°
   b. 180°
   c. 26°
   d. 154°

10.) \( \angle 1 \) and \( \angle 2 \) in this diagram are __________.
   a. Complementary
   b. Supplementary
   c. Congruent
   d. Vertical angle

11.) In the diagram, AD and AC are __________.
   a. Parallel
   b. Perpendicular
   c. Skew
   d. Intersection

12.) In the diagram, which lines must be parallel?
   a. J and k
   b. k and m
   c. l and m
   d. none of these

13.) Which statement is true?
   a. All quadrilaterals are squares.
   b. All quadrilaterals are rectangles.
   c. All parallelograms are rectangles.
   d. All rectangles are parallelograms.
14.) The two triangles shown are similar by __________.
   a. SAS Similarity
   b. ASA Similarity
   c. AA Similarity
   d. SSS Similarity

15.) Which of the following transformations is illustrated by the graph at the right?
   a. Dilation
   b. reflection in y = x
   c. translation
   d. reflection in the origin

16.) Parallelogram EFGH is similar to parallelogram WXYZ.

What is the length of \( \overline{WZ} \)?
   a. 3 in
   b. 6 in
   c. 7 in
   d. 9 in

17.) Inductive arguments differ from deductive arguments in that:
   a. the conclusion of a deductive argument is not certain.
   b. inductive arguments are more probable than deductive arguments.
   c. new information can weaken deductive arguments.
   d. inductive arguments can be highly probable but never certain.

18.) Which statement is not true of inductive and deductive arguments?
   a. Deductive arguments cannot become better or worse.
   b. Inductive arguments can become deductive arguments.
   c. Inductive arguments can become better or worse.
   d. Inductive and deductive arguments make different claims about their conclusions.

19.) Which terms may not be applied to deductive arguments?
   a. Weaker
   b. Valid
   c. Invalid
   d. Certain

20.) What is the contrapositive of "If it is Tuesday, then Marie has soccer practice?"
   a. If it is not Tuesday, then Marie does not have soccer practice.
   b. If Marie has soccer practice, then it is Tuesday.
   c. If Marie does not have soccer practice, then it is not Tuesday.
   d. Marie has soccer practice if and only if it is Tuesday.
Geometry

21.) Let $p$ be "there is lightning" and let $q$ be "we cannot go hiking." What is the converse of $p \rightarrow q$?

a. If there is lightning, then we cannot go hiking.
b. If we can go hiking, then there is no lightning.
c. If we cannot go hiking, then there is lightning.
d. If there is no lightning, then we can go hiking.
e. None of the above.